2025

$\boxed{\text{AB/BC 1}}$

a) $\frac{1}{4}\int_0^4 7.6 \arctan(0.2t)\,dt = 2.778$ acres

b) $\frac{C(4)-C(0)}{4-0} = C'(K)$

$1.282 = \frac{38}{25+K^2}$

$K = 2.154$ week

c) $\lim_{t\to\infty} \frac{38}{25+t^2} = 0$

d) $A'(t) = C'(t) - 0.1\ln(t)$

$C'(t) = (0.1)\ln(t)$

$t = 11.441$

t	$A(t)$
4	0.1545
11.441	7.317
36	1.743

$\longrightarrow$ Max value of 7.317 acres at $t = 11.441$ week

$\boxed{\text{AB 2}}$

a) $\int_0^3 g(x) - f(x)\, dx = 5.136$

b) $\text{Area} = x(g(x) - f(x))$

$V = \int_0^3 x(g(x) - f(x))\, dx = 7.704$

c) $\pi \int_0^3 (g(x)+2)^2 - (f(x)+2)^2\, dx$

d) $\frac{g(1) - g(0)}{1 - 0} = g'(c)$

$1 = 1 + \pi \cos(\pi c)$

$c = \frac{1}{2}$

BC 2

a) $\frac{dr}{d\theta} = 4\sin\theta\cos\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=1.3} = 4\sin(1.3)\cos(1.3)$

$= 1.031$

d) $\frac{dr}{dt} = \frac{\frac{dr}{d\theta}}{\frac{dt}{d\theta}} = \frac{4\sin\theta\cos\theta}{1/15}$

$\left.\frac{dr}{dt}\right|_{\theta=1.3} = 15.465$

b) $2\sin^2\theta = \frac{1}{2}$

$\theta = \pi/6, \frac{5\pi}{6}$

$\frac{1}{2}\int_{\pi/6}^{5\pi/6} (2\sin^2\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(\frac{1}{2}\right)^2 d\theta$

$= 2.066$

c) $4\sin\theta\cos^2\theta - 2\sin^3\theta = 0$

$\theta = 0.955$

θ	X
0	0
0.955	$r\cos\theta = 0.769$
$\pi/2$	0

$\theta = 0.769$

AB/BC 3

a) $R'(1) \approx \frac{100-90}{2-0} = 5$ words/min^2

b) R is differentiable, therefore continuous
There must be a time c on (0, 10) when c = 155
because R(0) = 90, R(10) = 162, and $90 < 155 < 162$

c) $\int_0^{10} R(t)\,dt \approx \left(\frac{90+100}{2}\right)(2) + \left(\frac{100+150}{2}\right)(6) + \left(\frac{150+162}{2}\right)(2)$

d) $\int_0^{10} -\frac{3}{10}t^2 + 8t + 100\,dt$

$-\frac{1}{10}t^3 + 4t^2 + 100t \Big|_0^{10}$

$= -100 + 400 + 1000 - 0 - 0 - 0$

$= 1300$ words

$\boxed{\text{AB/BC 4}}$

a) $g(x) = \int_6^x f(t)\,dt$

$g'(x) = f(x)$

$g'(8) = f(8) = 1$

b) g has a P.O.I. when

$g''(x) = f'(x)$ changes sign.

$f'(x)$ changes sign at turning points in $f(x)$.

$x = -3, 3, 6$

c) $g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2}(6)(3) = 9$

$g(0) = \int_6^0 f(t)\,dt = -\int_0^6 f(t)\,dt = -\frac{9\pi}{2}$

d) $g'(x) = f(x) = 0$

$x = -6, 0, 6, 12$

x	$g(x)$
-6	0
0	$-\frac{9\pi}{2}$
6	0
12	9

$x = 0$

AB 5

a) $V_H(t) = x_H'(t) = e^{t^2-4t}(2t-4)$

$V_H(1) = e^{1^2-4}(2(1)-4)$

$= e^{-3}(-2)$

$= \frac{-2}{e^3}$

b) $V_H(t) = 0$

$e^{t^2-4t}(2t-4) = 0$

$2t-4=0$

$t=2$

$V_J(t)=0$

$(2t)(t^2-1)^3=0$

$t=0,1,-1$

t		-1		0		1	
V_H	$-$	$-$	$-$	0	$+$	$+$	$+$
V_J	$-$	0	$+$	0	$-$	0	$+$

The particles are moving in opposite directions on $(-1,0)$ + $(0,1)$ because their velocities are opposite signs

c) $a_J(2) = V_J'(2) > 0$

$V_J(2) = 2(2)(4-1)^3$

$V_J(2) = (27)(4) > 0$

J is speeding up because $a_J(2)$ and $V_J(2)$ have the same sign

d) $x_J(0) + \int_0^2 V_J(t)\,dt = x_J(2)$

$7 + \int_0^2 2t(t^2-1)^3\,dt = x_J(2)$

$u = t^2-1$ $\quad u(2)=3$

$du = 2t\,dt$ $\quad u(0)=-1$

$7 + \int_{-1}^{3} u^3\,dt$

$7 + \frac{3^4}{4} - \frac{(-1)^4}{4} = x_J(2)$

$\frac{110}{64} = x_J(2)$

$\boxed{\text{BC 5}}$

a) $f''(x) = (3-x)(2)y\frac{dy}{dx} + (-1)y^2$

$f''(1) = (3-1)(2)(-1)\frac{dy}{dx} + (-1)(1)$

$f'(1) = (3-1)(-1)^2 = 2$

$f''(1) = -9$

b) $P_2(x) = -1 + 2(x-1) - \frac{9}{2!}(x-1)^2$

c) $\text{Error} \leq \frac{60}{3!}(1.1-1)^3$

$\leq 10(0.1)^3$

$\leq 10(0.001)$

≤ 0.001

d) $f(1.2) \approx -1 + f'(1)(1.2-1)$

$f(1.2) \approx -1 + 2(0.2) = -0.6$

$f(1.4) \approx -0.6 + (3-1.2)(-0.6)^2(1.4-1.2)$

$\boxed{\text{AB 6}}$

a) $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$

$3y^2y' - 2yy' - y' + \frac{1}{2}x = 0$

$y'(3y^2 - 2y - 1) = \frac{-x}{2}$

$y' = \frac{-x}{2(3y^2 - 2y - 1)}$

b) $y \approx -1 + \left.\frac{dy}{dx}\right|_{(2,-1)}(x-2)$

$y(1.6) \approx -1 + \left(\frac{-1.6}{2(3-2(-1)-1)}\right)(1.6-2)$

$y(1.6) \approx -0.92$

c) tangent is vertical when $2(3y^2 - 2y - 1) = 0$

$-x \neq 0$

$3y^2 - 2y - 1 = 0$

$(3y+1)(y-1) = 0$

$\cancel{y = -\frac{1}{3}}$ $y = 1$ $\rightarrow$ $1^3 - 1^2 - 1 + \frac{1}{4}x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$x = 2$

$(2, 1)$

d) $2xy + \ln y = 8$

$\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}[8)$

$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$

$2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{4}\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-6}{8 + 1/4}$

$= \frac{-24}{33}$

BC 6

a) $\lim_{n\to\infty}\left(\frac{(x-4)^{n+2}}{(n+2)3^{n+1}}\right)\left(\frac{(n+1)3^n}{(x-4)^{n+1}}\right)$

$\lim_{n\to\infty}\left(\frac{x-4}{3}\right)\left(\frac{n+1}{n+2}\right)=\frac{x-4}{3}$

$-1<\frac{x-4}{3}<1$

$-3<x-4<3$

$1<x<7$

$x=1$:

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}3^{n+1}}{(n+1)3^n}=3\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$

$\left|\frac{(-1)^{n+1}}{n+1}\right|=\frac{1}{n+1}$ is decreasing

$\lim_{n\to\infty}\frac{(-1)^{n+1}}{n+1}=0$

$3\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$ alternates

converges by AST

$x=7$:

$\sum_{n=1}^{\infty}\frac{3^{n+1}}{(n+1)3^n}=3\sum_{n=1}^{\infty}\frac{1}{n+1}$

IOC: $[1,7)$

comare to $\sum_{n=1}^{\infty}\frac{1}{n}$, which is the divergent harmonic series

$\lim_{n\to\infty}\left(\frac{1}{n+1}\right)\frac{n}{1}=1>0$

$\sum_{n=1}^{\infty}\frac{3^{n+1}}{(n+1)3^n}$ diverges by lim comparison to $\sum_{n=1}^{\infty}\frac{1}{n}$

B) $\frac{2}{6}(x-4)+\frac{3}{3}\frac{(x-4)^2}{3^2}+\frac{4}{4\cdot\cancel{3}^2}(x-4)^3$

c) $f'(x)=\sum_{n=1}^{\infty}\frac{\cancel{(n+1)}(x-4)^n}{\cancel{(n+1)}(3^n)}$

$=\sum_{n=1}^{\infty}\left(\frac{x-4}{3}\right)^n=\sum_{n=1}^{\infty}\underbrace{\left(\frac{x-4}{3}\right)}_{a}\left(\underbrace{\frac{x-4}{3}}_{r}\right)^{n-1}$

converges to:

$\frac{a}{1-r}=\frac{\frac{x-4}{3}}{1-\frac{x-4}{3}}$

$\frac{x-4}{3-(x-4)}=\frac{x-4}{7-x}$

d) No. The Taylor series for $f(x)$ centered at $x=4$ has an interval of convergence $[1,7)$.
The taylor series for $f'(x)$ centered at $x=4$ can only have fewer points in its interval of convergence. Since $x=8$ is outside of $[1,7)$, the series for $f'(x)$ cannot converge at $x=8$.